

# Pseudo-Exponential Current-to-Voltage Converter

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**Abstract**-This paper proposed a “pseudo-exponential” current-to-voltage converter with tunable input dynamic ranges. The proposed circuit uses CMOS transistors in saturation region. The feasibility of this circuit was verified through simulations at 1.5 V supply voltage, using 0.25  $\mu\text{m}$  CMOS process. The results show 30 dB range with the error less than  $\pm 0.5$  dB. The proposed circuit can be used for the design of an extremely low-voltage and low-power variable gain amplifier (VGA) and automatic gain control (AGC).

## 1. Introduction

Among the mostly used circuits in mixed signal VLSI circuits are VGAs and AGCs, which play an important role in telecommunications applications, medical equipment, hearing aid, disk drives and others [1-3]. The key component for the design of VGAs and AGCs is the exponential function generator. Unfortunately, there is no intrinsic logarithmic MOS device that operates in saturation region, which is necessary to design such generators. Therefore, to generate exponential characteristics, two following methods are used. The first one is based on a “pseudo-exponential” generator [1-4] in which the exponential characteristics can be approximated as

$$e^{2ax} \cong \frac{1+ax}{1-ax} \quad (1)$$

where  $a$  and  $x$  are the coefficient and the independent variable, respectively. The second method uses Taylor series expansion for realizing the exponential characteristics [5-9] in which the exponential characteristic can be expressed as

$$\exp(ax) = 1 + \frac{a}{1!}x + \frac{a^2}{2!}x^2 + \dots + \frac{a^n}{n!}x^n + \dots \quad (1^*)$$

The approximation equation, obtained by eliminating the higher order terms of Eq. (1<sup>\*</sup>) with small error for  $|ax| \ll 1$ , can be written as

$$\exp(ax) \cong 1 + \frac{a}{1!}x + \frac{a^2}{2!}x^2 = 1/2(1 + (1+ax)^2) \quad (2)$$

Also, the “pseudo-exponential” can be implemented as

$$e^{2ax} \cong \frac{1+ax+(ax)^2/2}{1-ax+(ax)^2/2} = \frac{1+(1+ax)^2}{1+(1-ax)^2} \quad (3)$$

Typically, the “pseudo-exponential” generator is of particular interest since it provides larger dB-linear range (about 15 dB with the error  $< \pm 0.5$  dB) compared to that of the other one. Moreover, the input range is symmetric. The comparison of these two methods is given in Fig. 1.

Obviously, the function,  $e^{2nax}$ , will have the output range multiplied by  $n$  times compared to that of the function  $e^{2ax}$  as shown in Fig.1.

This paper proposed an all CMOS exponential current-to-voltage converter for  $n = 2$ . The proposed circuit is based on the current-mode functional circuit [9], which has advantages that it can operate at extremely low-voltage applications, and its input voltage swing can be tunable easily. Moreover, the circuit is very independence on the temperature. The Simulation results will be given to verify the validity of this approach.

The organization of this papers is as follows: section 2 presents the current-mode functional circuit [9], section 3 proposes the completed circuit implementation based on section 2 and discusses the simulation results, and section 4 is the conclusions.

## 2. Current-mode functional Circuit design

The current-mode functional circuit in this section is adopted from [9] where all transistors are supposed to be in saturation region. Then the drain currents of transistors M5 and M6 in Fig. 2 (b) are given as

$$I_{d,M5,6} = K(V_{gs,M5,6} - V_t)^2 \quad (4)$$

It is shown from [10] that the output current,  $I_{out} = I_{d,M5} + I_{d,M6}$  of the circuit in Fig. 2 (b) can be given as

$$I_{out} = \frac{1}{2}K(V_2 - 2V_t)^2 + \frac{(I_{d,M6} - I_{d,M5})^2}{2K(V_2 - 2V_t)^2} \quad (5)$$

using the bias circuit as shown in Fig. 2 (a), the Eq. (5) can be written as

$$I_{out} = I_{d,M6} + I_{d,M5} = 2I_0 + \frac{I_{in}^2}{8I_0} \quad (6)$$

where the  $I_0$  is given as  $I_0 = (K/4)(V_2 - 2V_t)^2$  and  $I_{in} = I_{d,M6} - I_{d,M5}$  as shown in Fig. 2(b). In order to keep all devices in the ON state the input current should be in the range  $|I_{in}| < 4I_0$  [10].

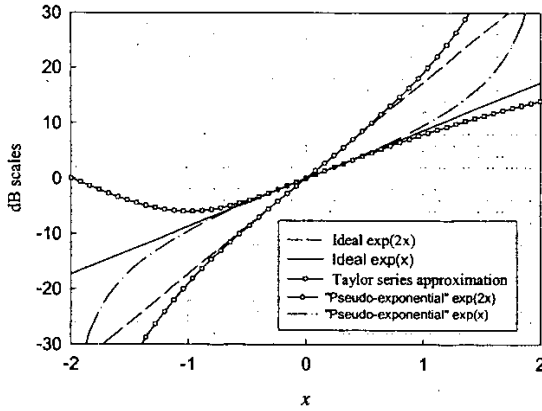


Fig. 1 Plots of various functions on dB-scale

The completed circuit implementations of the Eq. (5) is given in Fig. 3. It can be shown that the output current,  $I_{out}$ , in Fig. 3 can be written as

$$\begin{aligned} I_{out} &= 2I_0 + \frac{(I_{in} + k_1 I_0)^2}{8I_0} - k_2 I_0 \\ &= \frac{k_1^2 I_0}{8} \left[ \frac{8(2 - k_2)}{k_1^2} + \left( 1 + \frac{I_{in}}{k_1 I_0} \right)^2 \right] \end{aligned} \quad (7)$$

for  $k_1 = \sqrt{8(2 - k_2)}$  the exponential approximation in Eq.(2) is achieved and given as

$$I_{out} = \frac{k_1^2 I_0}{8} \exp\left(\frac{I_{in}}{k_1 I_0}\right) \quad (8)$$

The current,  $I_{out}$ , is thus an exponential approximation function of the input current  $I_{in}$ , where  $a = 1/(k_1 I_0)$ .

Consider the temperature stability of the circuit in Fig.3, as the temperature changes, assume that the bias current and the output current will be varied by  $I_0 + \Delta I_0$ , and  $I_{out} + \Delta I_{out}$  respectively. From Eq. (7) the following equation applies

$$\frac{I_{out} + \Delta I_{out}}{I_{out}} = \left( 1 + \frac{\Delta I_0}{I_0} \right) \exp\left[ \frac{-\Delta I_0 / I_0}{k_1 I_0 (1 + \Delta I_0 / I_0)} \right] \quad (9)$$

because  $\Delta I_0 / I_0 \ll 1$ , the Eq. (9) can be simplified as

$$\frac{I_{out} + \Delta I_{out}}{I_{out}} = \frac{(1 + \Delta M)}{\exp[\Delta M / k_1 I_0]} \quad (10)$$

where  $\Delta M = \Delta I_0 / I_0$ . From Eq. (10), it is obvious that while the temperature changes, both of the numerator and denominator of Eq. (10) are all increased or decreased, such that the circuit in Fig. 3 is very insensitive to the temperature.

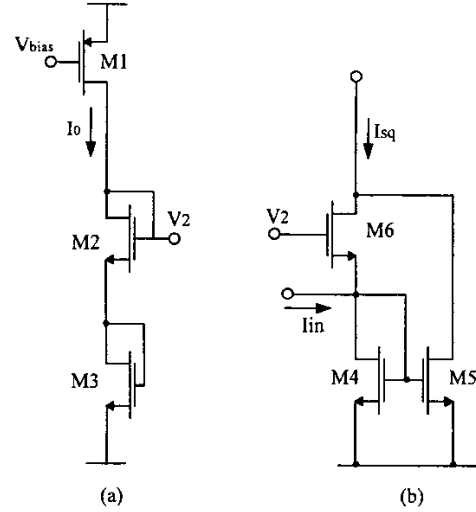


Fig. 2 (a) The bias circuit for the circuit in (b)  
(b) The current-mode building block for proposed exponential approximation.

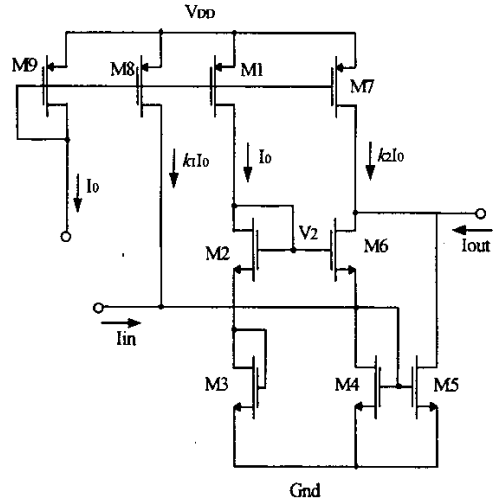


Fig. 3 The circuit implementation of the Eq. (2) based on the current-mode block shown in Fig. 2

### 3. The proposed “pseudo-exponential” current-to-voltage converter and simulation results

As presented earlier, the current-mode functional circuit has its transfer function shown in Eq. (7). For the inverse input current, the output current will be given as

$$I_{out1} = \frac{k_1^2 I_0}{8} \left[ \frac{8(2-k_2)}{k_1^2} + \left(1 - \frac{I_{in}}{k_1 I_0}\right)^2 \right] \cong \frac{k_1^2 I_0}{8} \exp\left(-\frac{I_{in}}{k_1 I_0}\right) \quad (11)$$

from Eq. (7) and (11), the division of these two currents leads to the following equation

$$I_{exp} = \frac{I_{out1}}{I_{out}} = \frac{\left[ \frac{8(2-k_2)}{k_1^2} + \left(1 - \frac{I_{in}}{k_1 I_0}\right)^2 \right]}{\left[ \frac{8(2-k_2)}{k_1^2} + \left(1 + \frac{I_{in}}{k_1 I_0}\right)^2 \right]} \quad (12)$$

for  $k_2 = 2$ , the Eq. (12) can be written as

$$I_{exp} = \frac{I_{out1}}{I_{out}} \cong \left(\frac{1-y}{1+y}\right)^2 \quad (13)$$

where  $y = I_{in}/(k_1 I_0)$ .

The circuit implementation of Eq. (13) is given in Fig. 4, where two identical current-mode functional circuits shown in Fig. 3 are used to generate two signals  $I_{out}$  and  $I_{out1}$ ,

respectively. As shown in Fig. 4, the current mirror M10-M11 is used to direct the current  $I_{out}$  to the drain of transistors M12 and M14, which operates in saturation region. The other current  $I_{out1}$  is also directed to the drain current of transistor M13, which is in triode region. A constant resistance,  $R_{in}$ , seen at the common drain node of the two diode connected transistors M12 and M14 can given as [4]

$$R_{in} = \frac{1}{K(V_{DD} - |V_{Tp}| - V_{Tn})} \quad (14)$$

this resistance converts the current  $I_{out}$  to a gate voltage of transistor M13,  $V_{G,M13}$ . Transistor M13 operates in the triode region and acts as a voltage controlled resistor. For small drain-source voltages, the resistance exhibited by M13 is  $R_{DS} \cong 1/K_{M13}(V_{G3} - V_{Tn})$ . Hence,  $I_{out1}$  flows through M13 and generates a drain-source voltage,  $V_{DS} = R_{DS} I_{out1}$ , proportional to  $I_{out1}/I_{out}$ .

The feasibility of this circuit was verified through simulations, using 0.25  $\mu\text{m}$  CMOS process at 1.5 V supply voltage. The simulation results are shown in Fig. 5, which shows 30 dB control range with the linearity error less than  $\pm 0.5$  dB. For different bias current  $I_0$ , the input dynamic ranges are adjusted. The solid curve in Fig.5 is given for  $I_0 = 40 \mu\text{A}$ . The proposed circuit consumes very low power that depends on the input dynamic range as shown in Table.1. For small input range, the  $k_2$  is slightly changed such that the dB output voltage ranges are reduced as shown in Fig. 5 by the dashed and dash-dotted curves.

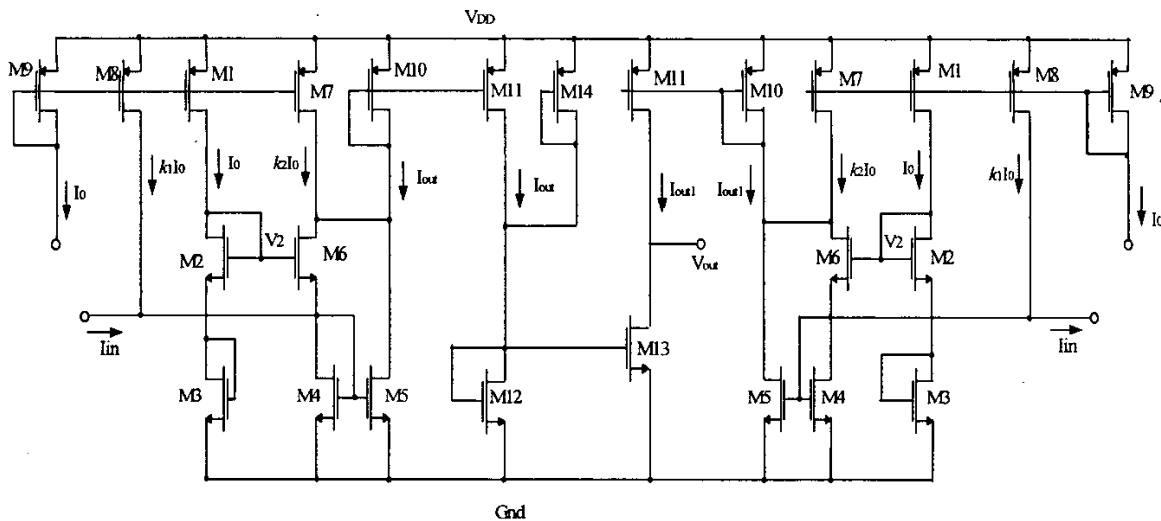


Fig. 4 The proposed “pseudo-exponential” current-to-voltage converter

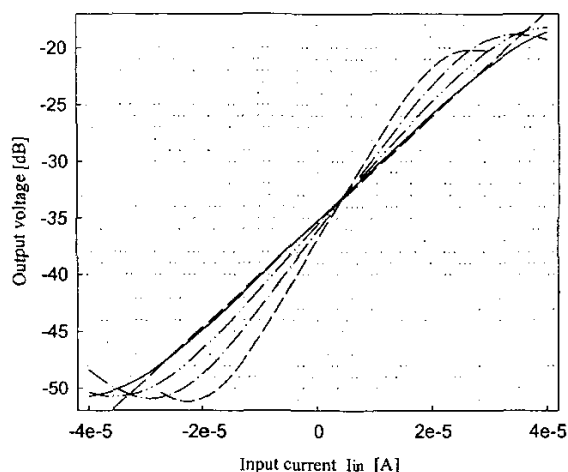


Fig. 5 The simulation results of the proposed circuit shown in Fig. 4 for various  $I_0$ .

TABLE. 1  
Power dissipation versus input dynamic range

| Input dynamic ranges [ $\mu$ A] | 20  | 30  | 40  | 50  | 60  |
|---------------------------------|-----|-----|-----|-----|-----|
| Power consumption [mW]          | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 |

#### 4. Conclusions

A novel approximation function to realize the exponential relation is presented at extremely low-voltage low-power applications. The average power consumption can be less than 0.1 mW at 1.5 V supply voltage. The proposed circuit has its input range tuned by the bias current  $I_0$ . Another advantage of this circuit is that it is very independence on the temperatures. The proposed V-I converter can achieve 30 dB output voltage range with the error less than  $\pm 0.5$  dB. The proposed circuit could be used in the design of an extremely low-voltage and low-power-VGA and AGC.

#### REFERENCES

- [1] R. Harijani, "A Low-power CMOS VGA for 50 Mb/s Disk Drive Read Channels," *IEEE Trans. Circuits and Syst.* vol. 42, no. 6, pp. 370-376, June, 1995.
- [2] K. M. Abdelfattah and A. M. Soliman, "Variable Gain Amplifier Based on a New Approximation Method to Realize the Exponential Function," *IEEE Trans. Circuits Syst.*, vol. 49, no. 9, Sep 2002.

- [3] H. Elwan, A. M. Soliman, and M. Ismail, "A. digitally controlled dB-linear CMOS variable gain amplifier," *Elect. Lett.*, vol. 35, no.20, pp. 1725-1727, 1999.
- [4] A. Motanemd, C. Hwang and M. Ismail, "CMOS exponential current-to-voltage converter," *Elect. Let.*, vol. 33, no. 12, pp. 998-1000, 5<sup>th</sup> June, 1997.
- [5] Quoc-Hoang Duong, T.Kien N, and Sang-Gug Lee, "Low-Voltage Low-Power High dB-Linear CMOS Exponential Function Generator Using Highly-linear V-I Converter," *IEEE International symposium on Low Power Electronics and Designs*, pp. 349-352 August 2003.
- [6] Cheng-Chieh Chang and Shen-luan Liu, "Current-mode pseudo-exponential circuit with tunable input range," *Electronic Letter*, vol. 36, no. 16, pp 1335-1336, 3<sup>rd</sup> August, 2000.
- [7] W. Liu, C. Chang, and S. Liu, "Realization of Exponential V-I Converter using composite NMOS transistors," *Elect. Let.*, vol. 36, no. 1, pp. 8-10, 6<sup>th</sup> Jan, 2000.
- [8] Quoc-Hoang Duong and Sang-Gug Lee, "A low-voltage, low-power, and high db-linear all cmos exponential function generator for AGC and VGA applications," will be presented at AMPC'03 Conference on November, South Korea.
- [9] Quoc-Hoang Duong and Sang-Gug Lee, "All CMOS Current-Mode Exponential Function Generator With Tunable Input Range," submitted to DATE 2004 conference, will be held in France.
- [10] K. Bult and H. Wallinga, "A Class of Analog CMOS Circuits Based on the Square-Law Characteristic of an MOS Transistor in Saturation," *IEEE J. of Solid-State Circuits*, vol. sc-22, no. 3, June 1987

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