Method of Computing Spectral Factors in Piecewise-Quadratic Bases and its Application in Problems of Digital Signal Processing

Olim Hidayov[1], Hakimjon Zaynuddinov[3], Hyung Chul Park[2], Kondekar Pravin[1], Sang-Gug Lee[1]
[1]Information and Communication University, Korea
[2]Hanbat National University
[3]Dongseo University
119, Mujiro, Yuseong-gu, Dae-jeon, Republic of Korea, 305-714, u-Radio Lab-514
Phone: (82) 42-866-6133 E-mail: olim_ah@yahoo.com

Keywords: signal, basis, spectrum, fast transformations

Abstract - Many different piecewise-quadratic bases are in existence, but they cannot be used practically since the method for calculation of factors in these bases have not been developed. This work is hardware-oriented method, that allows the use of existing algorithms of fast transformations based on Haar and Harmut's orthogonal step-function for the calculation of coefficients both piecewise-linear, and piecewise-quadratic bases factors.

1. INTRODUCTION

One of the basic features of orthogonal bases is the presence of fast algorithms for the definition of spectral factors. But these algorithms, as a rule, are developed for piecewise-constant bases. In existing work piecewise–polynomial bases have been shown, their advantages specified and the basic limitation has been shown to be as a result of the absence of a method for calculating the factors.

In this paper, we developed the method of calculating factors in the piecewise-quadratic bases using existing algorithm and good differential properties of splines. In other words, the requirements for algorithms of fast spectral transformation consist first of all in a minimality of operating quantity, simplicity of each operation and a minimality of the demanded volume of operative memory. Our developed method allows the reduction of quantity of arithmetic operations and volume of necessary memory. It increases speed because of using orthogonal bases for problems involving in digital signal processing [1] [2] [3] [4].

2. FAST ALGORITHMS CALCULATION FOR COEFFICIENTS IN VARIOUS BASES

We write down the formula of direct and return fast spectral transformations for sequence of readout of a signal $\{ x_i \}$ for any valid orthogonal piecewise-constant basis.

$$C_k = \frac{1}{2^p} \sum_{i=0}^{n-1} x(i) \cdot \varphi(k,i) \qquad (1)$$

$$X_i = \sum_{k=0}^{n-1} C_k \cdot \varphi(k,i) \qquad (2)$$

where k = Number of spectral coefficient, i = number of an element of sequence of the valid readout.

Fig.1: shows the graph of direct fast transformation of Haar (FHT) at N=16 proposed by Andrews.

Fig.1 shows the graph of direct fast transformation of Haar (FHT) at N=16. This graph has been proposed by Andrews.

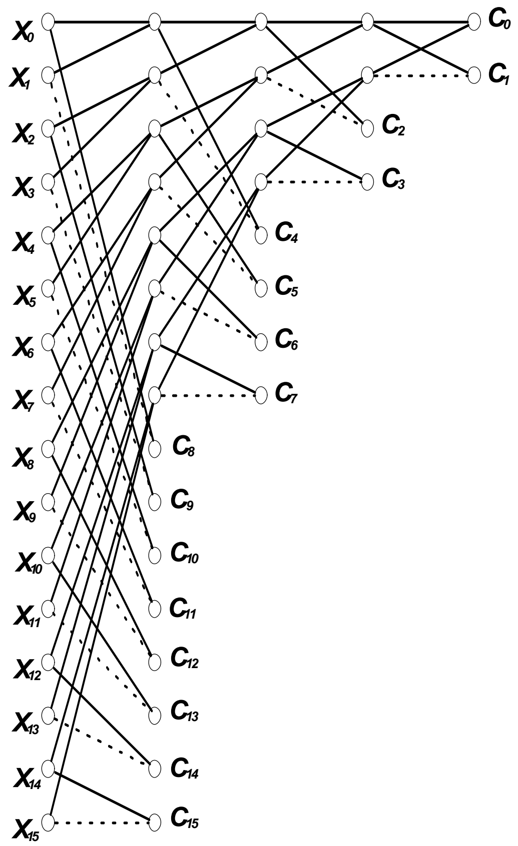

Fig 1.Fast transformation of Haar (FHT) at N=16 proposed by Andrews.

In this graph, the continuous lines correspond to operations of addition, while the hatch-lines are operations of subtraction. Entrance readout is denoted with X_0, X_1, ... , X_{15}, and results are denoted with C_0, C_1, C_2 ... , C_{15}.

The analysis of computational methods of factors in various bases has shown, that fast algorithms for calculation of factors exist only for piecewise-constant and piecewise-linear bases. Algorithms of calculation of factors in piecewise-quadratic bases have not been developed.

The analysis of Harmut matrix [4] helps to develop new algorithm of fast transformation. Factors C_0 and C_1 are decomposed with Harmut's basic functions. The zero and first order are defined respectively by the following formulas:

$$C_0 = \sum_{i=0}^{n-1} \Delta f_i; \quad (3)$$

$$C_1 = \sum_{i=0}^{n/2-1} \Delta f_i - \sum_{i=n/2} \Delta f_i; \quad (4)$$

For the second order of basic functions factors of Harmut's fast transformation C_2 and C_3 are calculated by grouping the sums of final differences under formulas:

$$C_2 = \left(\sum_{j=0}^{n/4-1} \Delta f_i - \sum_{j=n/4}^{n/2-1} \Delta f_j \right) - \left(\sum_{j=n/2}^{3n/4-1} \Delta f_i - \sum_{j=3n/4}^{n-1} \Delta f_j \right) \quad (5)$$

$$C_3 = \left(\sum_{j=0}^{n/4-1} \Delta f_j - \sum_{j=n/4}^{n/2-1} \Delta f_j \right) - \left(\sum_{j=n/2}^{3n/4-1} \Delta f_j - \sum_{j=3n/4}^{n-1} \Delta f_j \right) \quad (6)$$

Other factors for $P \geq 2$, $k \geq 4$ are calculated as the sum of a difference of a following view:

$$C_k = \left(\sum_{\{j\}} \Delta f j - \sum_{\{n/2^p + j\}} \Delta f j \right) + \left(\sum_{\{n/2^{p-1} + j\}} \Delta f j - \sum_{\{3n/2^p + j\}} \Delta f j \right) \quad (7)$$

$$C_k = \left(\sum_{\{j\}} \Delta fj - \sum_{\{n/2^p + j\}} \Delta fj \right) + \left(\sum_{\{n/2^{p-1} + j\}} \Delta fj - \sum_{\{3n/2^p + j\}} \Delta fj \right) \quad (8)$$

And the last on numbering factors equals:

$$\begin{aligned} C_{n-2} &= (\Delta f_{n-1} + \Delta f_{n-3}) - (\Delta f_{n-2} - \Delta f_{n-1}) \\ C_{n-1} &= (\Delta f_{n-4} - \Delta f_{n-3}) - (\Delta f_{n-2} - \Delta f_{n-1}) \end{aligned} \quad (9)$$

Let's consider approximating number on Harmut's piecewise-quadratic functions:

$$f(x) \cong \sum_{k=0}^{n-1} C_k \, hid_k(x) \quad (10)$$

Where hid = Double integration of Haar function.

The limitations of the given series shown the absence of fast algorithm of calculation of factors. This limitation can be overcome by application of a parabolic spline. If we take the second derivative of the parabolic spline interpolation on [0,1] function f (x), it will represent piecewise-constant function with changes of values of steps in units of a spline, and on piecewise -constant orthogonal basic functions. We shall write down, for example, decomposition derivative of a spline in Harmut's series:

$$S_2''(x) \cong \sum_{k=0}^{n-1} C_k \, hrm_k(x) \quad (11)$$

Where hrm = Harmut's function.

According to theorems of the limited convergence and of integration of the closed systems as a result of integration of both parts we will get:

$$S_2'(x) = 2^p \int_0^x S_2''(u)\,du = \sum_{k=0}^{n-1} C_k \, hin_k(x) + S_2'(0) \quad (12)$$

$$S_2(x) = \int_0^x S_2'(u)\,du + S_2(0) = \sum_{k=0}^{n-1} C_k \, hid_k(x) + S_2'(0) + S_2(0) \quad (13)$$

Thus, it follows that factors of decomposition in a series in Harmut's orthogonal functions takes the second derivative of the parabolic spline interpolating function in binary-rational units are factors of decomposition of the first derivative of a spline on hin-functions, and the spline on hid-functions. The factor at a linear part of decomposition is defined as value of the first derivative of spline $S_2(x)$ at the point $x = 0$, and constant component as a value of a spline at that point.

One of the major properties the spline of functions is the presence of high degree derivatives [5] [6] [7] [8]. This property allows developing the hardware-oriented algorithm for calculation of factors in piecewise-quadratic bases which consists of following items:

Input of initial functional dependence, i.e. input array of real experimental data.

To define b-factors under the formula

$$b_i = \frac{1}{8}\left(- f_{i-1} + 10 f_i - f_{i+1}\right) \quad (14)$$

For calculate values of an approximating spline $S_2(x)$ under the formula

$$f(x) \cong S_2(x) = b_{-1} \cdot B_{-1}(x) + b_0 \cdot B_0(x) + b_1 B_1(x) \quad (15)$$

Thus instead of values of basic functions $B_i(x)$, the derivatives of the second order are used on a line segment [-1,5;-0,5]- $B''(x) = 1$, on a line segment [-0,5; 0,5] - $B''(x) = 2$ and on a line segment [0,5; 1,5]- $B''(x) = 1$.

$$f''(x) \cong S_2''(x) = b_{-1} \cdot B''_{-1}(x) + b_0 \cdot B''_0(x) + b_1 B_1''(x) \quad (16)$$

As a result of these calculations, the array of values of the second derivative of an approximating $S''_2(x_i)$ spline is achived.

3. To form array $S''_2(x_i)$
4. Above elements of the received array are used to execute fast transformations by Andrew's graph "*Fig.1*". and to define coefficients. These coefficients are coefficients of piecewise-quadratic basis.

3. EXPERIMENTAL RESULTS

Series of numerical experiments have been carried out on the research of piecewise-quadratic bases. With use of the offered algorithm of calculation of coefficients in Haar and Harmut's piecewise-quadratic bases, "*Table-1*" is achieved. Here the factor of compression K_c is defined by the formula:

$$K_c=N_1(N-N_1), \quad (17)$$

Where N = Quantity of readout of function
N_1 = Quantity of the zero factors received as a result of use of offered algorithm.

№	Function	Haar's Fast transformation	
		K_c	Zero coefficients
1	$Y=e^x$	2.06	51.5%
2	Y=e-2x·cos(4πx)	1.09	10.9%
3	Y=arcth(x)	2.5	60.1%
4	ABT	1.05	5.5%
5	AME	1.07	6.3%

Table 1(a): Results of research of fast transformations Haar in piecewise-quadratic bases.

№	Function	Harmut's Fast transformation	
		K_c	Zero coefficients
1	$Y=e^x$	3.76	73.4%
2	Y=e-2x·cos(4πx)	2.28	56.2%
3	Y=arcth(x)	3.46	71.0%
4	ABT	1.21	17.2%
5	AME	1.29	24.2%

Table 1(b): Results of research of fast transformations Harmut in piecewise-quadratic.

Here:
ABT- Array received as a result of bench tests;
AME -Array received on magnetic exploration

The numerical experiments allow us to draw a conclusion that the number of zero coefficients at digital processing of signals received as a result of bench tests in Haar and Harmut's piecewise-quadratic bases ranges from 5 % up to 17 %, when processing the geophysical signals received as a result magnetic exploration we get values ranging from 5 % up to 25 %, and while processing elementary functions (and also functions consisting of their combinations) this parameter gives us value from 10 % up to 70 % with an accuracy of 10-4-10-6. It is established, that decomposition (3) allows receiving high speed in Haar's basis and the big factor of compression in Harmut's basis. Also as a result of researches it is revealed, that with increase in quantity of readout function N, the values of factors decreases on exponential law.

4. SUMMARY

Haar and Harmut's piecewise-linear and piecewise-quadratic bases can be relatively achieved from a single and double integral of piecewise-constant bases. The derived bases have higher accuracy than piecewise-constant bases. As a result of using good differential qualities of splines, which are derived from a method that allows the use existing algorithms of fast transformation for piecewise-linear and piecewise-quadratic bases. The qualities of the bases have been researched on above with the use of numerical experiments.

5. CONCLUSION

As a result of research on methods of approximating functional dependence shows their limitation as weak convergence, discontinuity, rather low accuracy of approximation, necessity of great volume of memory for factors are revealed. In order to overcome these limitations, the necessity for transition to piecewise-quadratic bases was shown. Advantages of piecewise-quadratic bases: greater accuracy and good smoothness of approximation in comparison with piecewise-constant and piecewise-linear bases. The limitation of piecewise quadratic bases shows absence of fast algorithms for calculating coefficients. In order to overcome this limitation in given work, the method of calculation coefficients in Haar and Harmut's piecewise-quadratic bases was proposed. The method is based on applications of good differential properties of basic splines, it is hardware-focused and allows to use existing algorithms of fast transformations in bases of orthogonal piecewise-constant functions for calculation of factors both piecewise-linear and piecewise-quadratic bases.

ACKNOWLEDGMENT

This work was supported by the Korea Science and Engineering Foundation(KOSEF) through the National Research Lab. Program funded by the Ministry of Science and Technology (No. R0A-2007-000-10050-0)

REFERENCES

[1] Bohan K.A., Koroleva N.A. Estimation parameters of efficiency two-dimensional transformations Haar // Electronic modelling. 2003. №5. C.45 -56.

[2] Richard E. Blahut. Fast Algorithms for Digital Signal Processing. 1989, -448p.

[3] Zalmanzon L.A. Transformation Fure, Uolsh, Haar and their applications in management,

communications and other areas. - M.: the Science, 1989. - 496 p.
[4] Kasimov S.S., Zaynidinov H.N. Application of splines for expansion of opportunities of fast algorithms. // the Uzbek magazine of a problem of computer science and power. 2002. -№ 6, C. 7-12.
[5] Castrillon-Canda J., Amaratunga K. Fast Estimation of Continuous Karhunen-Loeve Eigenfunctions Using Wavelets // IEEE Trans. On Signal Processes. – V.50. – 2002. – No.1. – pp.78-86.
[6] Chang-Kun Song, Kyung-Seok Kim. Efficient Signal Feature Detection using Spectral Correlation Function in the Fading channel. International Journal of Contents, vol.3, № 2, 2007, pp. 35-39
[7] A. Boggess and F.J. Narcowich, A First course in Wavelets with Foureir Analisis, Prentice-Hall, 2001.
[8] Ignacio Arganda-Carreras, Carlos O. S. Sorzano, Roberto Marabini, Jose M. Carazo, Carlos Ortiz de Solorzano, and Jan Kybic, "Consistent and Elastic Registration of Histological Sections using Vector-Spline Regularization", Lecture Notes in Computer Science, Springer Berlin / Heidelberg, volume 4241/2006, CVAMIA: Computer Vision Approaches to Medical Image Analysis, pages 85-95, 2006