Scalable Small-Signal Model for BJT Self-Heating

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Abstract—**The effects of self-heating on BJT behavior are demonstrated through measurement and simulation. Most affected are the small-signal parameters Y_{22} and Y_{12}. A frequency-domain solution to the heat-flow equation is presented that applies to any rectangular emitter geometry. This model, although simple enough for CAD, predicts thermal spreading impedance with good accuracy for a wide range of frequencies.**

I. Introduction

It HAS LONG been known [1], [2] that collector-to-emitter thermal feedback can profoundly affect bipolar transistor performance. This effect is becoming more important as current densities and thermal spreading impedances rise with technology scaling. Previously developed thermal-impedance models were based in the time domain [3]. It was shown in [4] that neglecting thermal effects caused errors up to 4% in the predicted delay of a simple BJT logic gate. This letter will show that local heating causes much larger errors in BJT small-signal parameters. A new model is derived that predicts thermal corrections to BJT small-signal parameters based on a solution of the heat-flow equation in the frequency domain. The model is applicable in principle to any emitter geometry. Polynomial approximations provide simplified expressions for rectangular emitters. These expressions are simple enough for compact modeling in SPICE.

If a unit impulse of heat occurs instantaneously at some point in a uniform medium, the temperature rise at a point a distance r away is given by [5]

$$T(r, t) = \frac{1}{8\rho c(\pi \kappa t)^{3/2}} \exp(-r^2/4\kappa t) \tag{1}$$

where ρ is the density, c is the specific heat, and κ is thermal diffusivity. An identical image source can be added to create an adiabatic surface at the plane equidistant between the sources. In their time-domain derivation, Joy and Schlig [3] integrated (1) over the volume of the collector space-charge region (SCR) and its image. The result is a closed-form expression for the impulse response at any point in the semiconductor. The time integral of this expression gives the response to a unit step in the collector power. The integral is complicated and does not have a closed-form solution. The step response at $t = \infty$ gives the thermal resistance R_{th}, defined as the total temperature rise caused by a unit step in the collector power.

A frequency-domain model can be derived by first taking the Laplace transform of (1), yielding

$$Z_{th}(r, s) = \frac{1}{4\pi K r} \exp\left(-r\sqrt{\frac{s}{\kappa}}\right) \tag{2}$$

where $K = \rho\kappa c$ is thermal conductivity. For $s = 0$, this equation gives R_{th} for the point source. R_{th} can be integrated over the collector SCR and its image to yield the thermal resistance at any point $r' = (x', y', z')$ for arbitrary geometry. In rectangular coordinates

$$R_{th}(W, L, H, D, r') = \frac{1}{4\pi K} \int_0^W \int_0^L \left[\int_D^{D+H} \frac{dx\,dy\,dz}{r} + \int_{-D}^{-(D+H)} \frac{dx\,dy\,dz}{r}\right] \tag{3}$$

where $r = \sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}$, W and L are the width and length of the emitter, D is the depth of the base–collector junction, and H is the SCR width. This integral is not solvable in closed form, so least-squares fitting was used to generate the following approximation to (3), in which r' is taken as the point at the surface above a corner of the emitter:

$$f_1(d, h) = (0.058\, d + 0.14)h + 0.34\, d + 0.28 \tag{4a}$$

$$f_2(a) = 0.98 + 0.043\, a - (6.9 \cdot 10^{-4})a^2 + (3.9 \cdot 10^{-6})a^3 \tag{4b}$$

$$r_{\text{eff}} = 2\sqrt{WL}\, f_1 f_2 \tag{4c}$$

$$R_{th} = 1/(2\pi K R_{\text{eff}}) \tag{4d}$$

where $d = D/\sqrt{WL}$, $h = H/\sqrt{WL}$, and $a = W/L$. The approximation agrees with numerical solution of (3) to within 5% for all reasonable geometries. To find R_{th} for any other point r' at the surface above the emitter, the transistor can be broken into four smaller rectangular transistors, meeting at r'. Equation (4) can then be used with modified values for W and L to compute R_{th} for each quadrant. The overall value for R_{th} can be found using

$$R_{th}(r') = \frac{1}{WL} \sum_{i=1}^{4} R_{thi} \cdot W_i \cdot L_i. \tag{5}$$

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Table I gives calculated R_{th} values for r' located in the center of the emitter and at the corner for several transistor geometries. Values for H were computed using the step-junction approximation. We believe this is the first compact model that can predict the variations of R_{th} across the emitter and the variations with D and H.

II. Small-Signal Model

Mueller [6] showed that BJT common-emitter y parameters can be corrected for thermal feedback using

$$Y_{mn} = \frac{Y_{mn}e + D_m Z_{th} I_m I_n}{1 - D_m Z_{th} P} \tag{6}$$

where m or $n = 1$ for the base or 2 for the collector, Y_{mnE} is the uncorrected electrical Y parameter, Z_{th} is the thermal impedance, and P is the dissipated power. D_2 is the fractional temperature coefficient of the collector current (typically about 7%/K) and D_1 is that of the base current, equal to $D_2 - D_\beta$, where D_β is the fractional temperature coefficient of β (typically around 0.7%/K). Equation (6) can be used with $Z_{th} = R_{th}$ to thermally correct the dc small-signal parameters g_{mn}.

Fig. 1 shows measured and simulated values for the normalized output resistance parameter $V_A = I_C/g_{22} - V_{CE}$ for a 23 × 23 μm^2 silicon BJT. Note that most circuit models use a constant value (the Early voltage) for this parameter. Measured g_{22} variations can be used in (6) to extract a best-fit value for R_{th}. The last column in Table I compares R_{th} values found by this method to calculated values. Note that most of the measured values match well with the calculated R_{th} for the corner of the emitter. This result appears to be largely coincidental, but it is fortunate, as it means that a single effective temperature can be assigned to the emitter using (4) with good accuracy. The last row in Table I, corresponding to the narrowest emitter, shows an error in R_{th} greater than 35%. This error may be caused by multidimensional current flow, which would lead to nonuniform power dissipation; measurements on other small devices will be needed to determine whether this reflects a limit on the model's scalability.

The other dc small-signal parameter which is strongly affected by thermal feedback is g_{12}. Electrical models predict only a small negative value for this parameter, caused by Early-effect modulation of recombination in the quasi-neutral base. This effect is normally neglected. Equation (6) predicts a substantial positive value for g_{12} approaching g_{22}/β with increasing I_C. This can have a substantial effect on circuit performance, as the input and output impedances become strongly dependent on the loading at the opposite port. Note that (6) shows that errors in small-signal parameters due to neglecting thermal feedback can occur without large dc power dissipation.

III. AC Modeling

Equation (2) can be used to model thermal spreading impedance above dc if the r_{eff} from (4) is substituted for r in (2). The results are compared to the inverse Fourier transform of Joy and Schlig's predicted impulse response in Fig. 2. For this case (the same device as in Fig. 1) the magnitudes differ by less than 6% for frequencies up to $f_{pk} = 350$ kHz, defined as $1/(2\pi t_{pk})$, where $t_{pk} = r_{\text{eff}}^2/(6\text{ K}) = 0.45$ μs is the time at which the impulse response in (1) reaches a maximum. At this frequency $|Z_{th}|$ has dropped to 18% of R_{th}. Note that the commonly used one-pole approximation [4], [7], [8] predicts most of the drop in the $|Z_{th}|$ to occur over one decade of frequency, whereas the $|Z_{th}|$ in Fig. 2 varies over three decades of frequency.

The phase response is less accurate, differing by more than 30° for frequencies greater than f_{pk}. These phase errors are easily understood: the model represents the transistor as a point-source collector and a point emitter, separated by an r_{eff} which is inevitably greater than the base width. Thus, the model overestimates the initial delay. However, since they coincide with decreasing $|Z_{th}|$, these phase errors cause relatively little error in the y parameters.

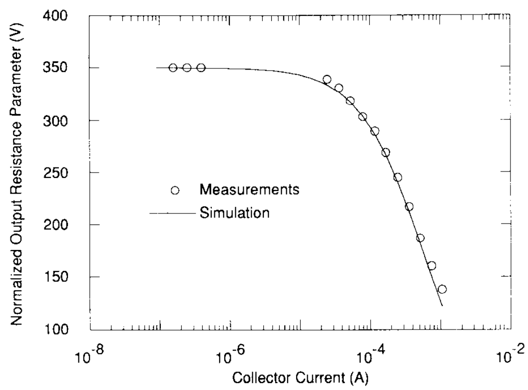

Fig. 1. Measured and simulated values for the normalized resistance output parameter $V_A = I_C/g_{22} - V_{CE}$ for a silicon BJT with 23 × 23-μm^2 emitter, collector junction depth of 3.0 μm, and epi doping of 10^{15} cm^{-3}. The value for H in (4) was 2.8 μm, calculated assuming a step junction with $V_{CE} = 6$ V, which led to $R_{th} = 70$ K/W. The simulation used $g_{22E} = I_C/V_{AE}$, where $V_{AE} = 350$ V is the electrical Early voltage, measured at low currents. The data were corrected to account for package thermal impedance.

TABLE I
Calculated and Measured Thermal Resistance for Several Transistor Geometries. Calculations Were Based on (4) and (5). Measured Values Give Best Fits to Output Resistance Using (6) and Include an Estimate of Experimental Error

Type	W (μm)	L (μm)	R_{th} at corner	R_{th} at center	Measured R_{th}
a	23	23	69	115	74 ± 5
	23	83	36	65	41 ± 4
	12	12	112	170	118 ± 10
b	4	2	492	778	480 ± 20
	8	2	348	582	390 ± 5
	10	2	306	520	350 ± 20
	10	4	251	443	290 ± 10
c	7.2	1.2	467	814	340 ± 75

a: $D = 3$ μm, $N_{\text{epi}} = 1 \times 10^{15}$ cm^{-3}, $V_{CE} = 6$ V, $H = 2.83$ μm.
b: $D = 0.4$ μm, $N_{\text{epi}} = 7 \times 10^{15}$ cm^{-3}, $V_{CE} = 3$ V, $H = 0.85$ μm.
c: $D = 0.25$ μm, $N_{\text{epi}} = 2 \times 10^{16}$ cm^{-3}, $V_{CE} = 2$ V, $H = 0.35$ μm.

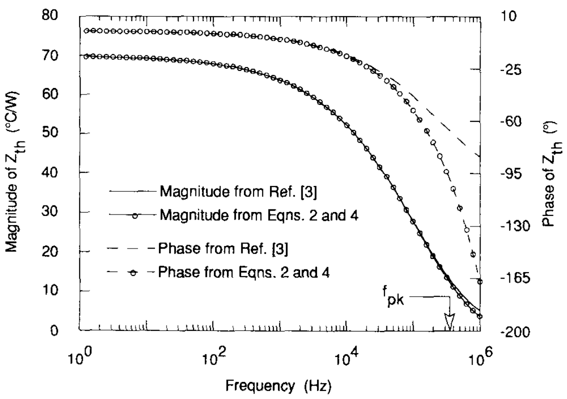

Fig. 2. Magnitude and phase of thermal spreading impedance Z_{th}. Circled values were computed using (2) and (4); uncircled curves were computed using the Fourier transform of the model in [3].

IV. Conclusion

The model described above provides a simple and practical way to improve the accuracy of BJT circuit simulation. Polynomial approximations are used to express the variation of R_{th} across the emitter; the results are equivalent to the infinite integral in [3]. Measurements show that a single value of R_{th} can represent typical BJT thermal behavior at dc. A frequency-domain solution for Z_{th} is presented which is more accurate than the commonly used single-pole thermal-impedance model. The small-signal model has already been implemented in SPICE. Later the modeling will be extended to include dc and transient analyses. Also planned are extensions of the model to other device geometries, especially multiple-emitter stripes.

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