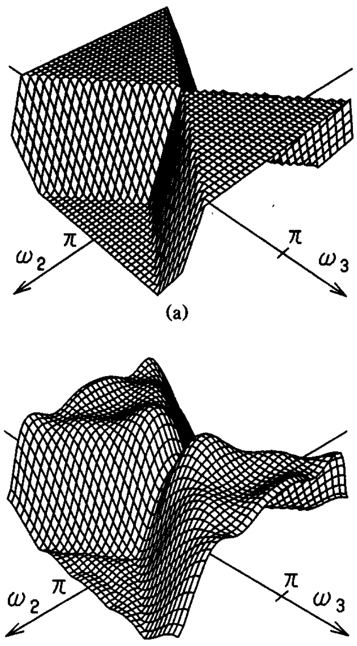

(b)

Fig. 2. (a) Magnitude response specification of cone filter ($\omega_1 = 0$). (b) Resultant magnitude response of cone filter ($\omega_1 = 0$).

VII. Numerical Example

A numerical example is shown for the frequency-domain design. The frequency response of a cone filter [2] is the following:

$$H_d(\omega_1, \omega_2, \omega_3) = \begin{cases} 1 & \text{if } \omega_r \leqslant 0.8\omega_3 \\ 1 - \dfrac{\omega_r - 0.8\omega_3}{0.4\pi} & \text{if } 0.8\omega_3 < \omega_r < 0.8\omega_3 + 0.4\pi \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

$$\omega_r = \left(\omega_1^2 + \omega_2^2\right)^{1/2}. \quad (24)$$

The magnitude response $A_d(l,m,n)$ on sampled points is produced from the frequency response by (9) with $L = M = N = 21$. It is decomposed by the outer product expansion, and the first four singular values are retained and the others are truncated. The specifications of 1-D digital filters are approximated to minimize the l_2 norm of the error of the magnitude response by the Davidon–Fletcher–Powell method. The filter orders are second for $\phi_p(z_1)$ and $\chi_p(z_2)$ $(p = 1,2,3,4)$, and third for $\psi_p(z_3)$ $(p = 1,2,3,4)$, respectively.

The magnitude responses of the specification and the approximation result on the plane with $\omega_1 = 0$ are shown in Fig. 2(a) and (b). The approximation error is 13.16% for the following criterion:

$$E_2 = \frac{\left[\sum_{l,m,n} \left(A_d(l,m,n) - A'(l,m,n)\right)^2\right]^{1/2}}{\left[\sum_{l,m,n} \left(A_d(l,m,n)\right)^2\right]^{1/2}}. \quad (25)$$

VII. Conclusion

This paper has proposed the efficient design method of 3-D digital filters by the outer product expansion. It can decompose design problems of 3-D digital filters into design problems of 1-D digital filters by the outer product expansion. Both space domain specifications and frequency domain specifications can be designed by this method. Diagonal symmetries of 3-D digital filters can be exploited to reduce computations in the design procedure. Moreover, the parallel separable structure produced by this method has high parallelism, regularity, and modularity, and so it is suitable for parallel and VLSI implementation of 3-D digital filters.

Acknowledgment

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Extension of the Open-Circuit Time-Constant Method to Allow for Transcapacitances

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***Abstract*—This paper reviews a previously published method for determining frequency-domain transfer functions of linear circuits and extends the method to allow for transcapacitors. The method is an extension of the familiar open-circuit time constant analysis technique, which depends on successive analyses of frequency-independent circuits. Where the original technique required finding the resistance "seen" by each capacitor, the extended technique requires finding a transresis-**

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tance for each transcapacitor. This paper presents a derivation of the generalized time-constant technique and demonstrates its application in a simple circuit.

I. Introduction

The earliest small-signal models for bipolar and field-effect transistors included only resistors, capacitors, and frequency-independent controlled sources. These models were simple and intuitive, and a number of useful "tricks" were available to analyze circuits based on them. However, several workers [1]–[3] pointed out inaccuracies due to these representations; more accurate models were then developed which used stored transistor charges as state variables.

A key feature of such charge-based models is that the terminal currents include terms of the form $C_{jk}(dV_k/dt)$, where C_{jk} is a constant and V_k is the instantaneous voltage at port k. Such a term can be represented in an equivalent circuit as a voltage-controlled current source across port j. In the small-signal frequency domain, such a source takes on the value $sC_{jk}V_k$. C_{jk} has units of capacitance, but for $j \neq k$, the controlled source represents not a capacitor but a *transcapacitor*. A capacitor can only be used when $j = k$. In general, transcapacitances are nonreciprocal, meaning $C_{jk} \neq C_{kj}$. The need for nonreciprocal elements to model active devices is not surprising—a transcapacitor is related to a capacitor the same way a transconductance is to a conductance.

A number of small-signal models including transcapacitors has been implemented in circuit simulators such as SPICE. These include the BSIM MOSFET model [4], the bipolar transistor model in the device/circuit simulator MMSPICE [5], and the five-terminal silicon-on-insulator model of Fossum and Veeraraghavan [6]. Typically, these models are accurate to frequencies about three times higher than those using only reciprocal capacitors. One would thus expect these models to be popular with circuit designers. In fact, some designers have resisted using these models, in part because some of the familiar circuit analysis techniques cannot be applied.

In particular, the open-circuit time-constant method [7] for estimating dominant pole frequencies has not been applicable to circuits with transcapacitors. The present work demonstrates how this simple method can be extended to such circuits. Furthermore, this paper shows how to handle transcapacitors in an extension of the time-constant technique [8], which allows simplified calculation of complete transfer functions. It uses successive analyses of resistive networks to find dc driving-point and transfer functions, requiring no complex algebra or frequency-dependent terms.

The extended time-constant method is derived in Section II. The derivation is an extension of the development in [7]. This method could be extended further to allow for inductances and for other frequency-dependent controlled sources, but such extensions are excluded here. Section III gives an example of use of the method.

II. Derivation of the Extended Time-Constant Method

Consider a multiport network, with a port defined for each capacitor, for each transcapacitor, and for each voltage which controls a transcapacitor (a transcapacitive controlling voltage, or TCV), as well as ports for the output (port o) and for the input (port i). Now define Δ as the determinant of the short-circuit admittances y_{jk}. It is convenient to define N as one less than the number of ports. Δ has the form

TABLE I
Admittance Cofactor Representation for Various Transfer Functions

Type of Function		Cofactor Expression Δ_N/Δ_D
Voltage Gain:	$\frac{V_o}{V_i}$	$\frac{\Delta_{io}}{\Delta_{ii}}$
Current Gain:	$\frac{I_o}{I_i}$	$\frac{\Delta_{io}}{\Delta_{oo}}$
Transimpedance:	$\frac{V_o}{I_i}$	$\frac{\Delta_{io}}{\Delta}$
Transadmittance:	$\frac{I_o}{V_i}$	$\frac{\Delta_{io}}{\Delta_{ii,oo}}$

$$\Delta = \begin{vmatrix} g_{11}+sC_{11} & g_{12}+sC_{12} & \cdots & g_{1,N+1}+sC_{1,N+1} \\ g_{21}+sC_{21} & g_{22}+sC_{22} & \cdots & g_{2,N+1}+sC_{2,N+1} \\ \vdots & \vdots & \ddots & \vdots \\ g_{N+1,1}+sC_{N+1,1} & g_{N+1,1}+sC_{N+1,1} & & g_{N+1,N+1}+sC_{N+1,N+1} \end{vmatrix}. \quad (1)$$

C_{jj} is the value of the capacitor (if any) at port j, and C_{jk}, $j \neq k$, is the transcapacitance from a TCV at port k that controls a transcapacitor at port j. No capacitive element occurs in more than one y_{jk}. In most circuits most of the possible coefficients C_{jk} are zero.

All possible transfer functions can be expressed as ratios of cofactors formed by deleting certain rows and columns of Δ. It is not necessary in practice to actually form these determinants; they are used here only for the purposes of this derivation. The numerator cofactor Δ_N in each case is Δ_{io}, the Nth-order determinant formed by deleting row i for the input and column o for the output. All terms y_{jk} having $j = i$ or $k = o$ are thereby deleted. Such deleted terms correspond to self-admittances at the input and output ports, and reverse transadmittances from the output back to some other port and from any port back to the input. It is useful to define $\boldsymbol{C}_N$ as the set of all nonzero capacitive elements C_{jk} having $j \neq o$ and $k \neq i$, corresponding to the coefficients of s in Δ_N.

The denominator determinant Δ_D is formed in a similar way. Selection of rows and columns to be deleted depends on the type of transfer function to be computed. Simply, any port where the short-circuit output current is defined or where the input voltage is applied is considered as a short circuit in finding Δ_D, and its corresponding row and column are deleted. The various possibilities are summarized in Table I. Δ_D is the cofactor of Δ with row and column deleted for any shorted port. The nonzero coefficients of s in Δ_D form the set $\boldsymbol{C}_D$: the set of all nonzero capacitive elements C_{jk} for which neither j nor k corresponds to a shorted port.

When expanded, Δ_D has the form

$$\Delta_D = b_0 + b_1 s + b_2 s^2 + \cdots + b_{N_d} s^{N_d} \quad (2)$$

where N_d is the order of Δ_D. N_d equals N for a voltage gain or a current gain, $N-1$ for a transadmittance, or $N+1$ for a transimpedance. Now, $b_0 = \Delta_D^0$, the determinant evaluated with all capacitive elements set to zero. It is useful to write

$$\frac{\Delta_D}{\Delta_D^0} = 1 + a_1 s + a_2 s^2 + \cdots + a_{N_d} s^{N_d} \quad (3)$$

where $a_j = b_j/\Delta_D^0$.

Expanding the determinant shows that the first-order term is

$$a_1 = \sum_{C_D} \left[C_{jk} \frac{\Delta_{Djk}^0}{\Delta_D^0} \right] \quad (4)$$

where the summation over C_D implies inclusion of a term for each element in C_D. From Cramer's Rule, $\Delta_{jk}/\Delta_D^0 = R_{kj}^0$, the transresistance from a current source at port j to an open-circuit voltage at port k, with all capacitive elements set to zero. For a capacitor C_{jj}, the corresponding element R_{jj}^0 is just the driving-point resistance at port j. These resistive coefficients can be calculated from straightforward circuit analysis, so the determinants themselves are not needed. Since the sum of these RC products forms a conservative estimate of the circuit's dominant time constant, this method is often called "time-constant analysis."

Calculation of the second-order coefficient a_2 is more complex. Second-order terms involve products of pairs of capacitive elements. Let C_{D2} denote the set of all unique products of pairs $C_{jk}C_{lm}$ of nonzero capacitive elements in C_D such that $j \neq l$ and $k \neq m$. This set contains all the pairs except those where a capacitor and a transcapacitor, or two transcapacitors, exist at the same port (coefficients in the same row) or a capacitor and a TCV, or two TCV's (same column), exist at the same port. Each element in this set is multiplied by a corresponding coefficient

$$a_{jk,lm} = \frac{\Delta_{Djk,lm}^0}{\Delta_D^0}. \qquad (5)$$

For $\Delta_{Djk}^0 \neq 0$, this can be expanded as

$$\frac{\Delta_{Djk,lm}^0}{\Delta_D^0} = \left[\frac{\Delta_{Djk,lm}^0}{\Delta_{Djk}^0}\right]\left[\frac{\Delta_{Djk}^0}{\Delta_D^0}\right]. \qquad (6)$$

Now $\Delta_{Djk}^0/\Delta_D^0$ is just R_{kj}^0, as shown previously, but the first factor's meaning has not yet been established. Note that for $g_{jk} \neq 0$

$$\Delta_{Djk}^0 = \frac{\Delta_D^0}{g_{jk}} - \frac{\Delta_D^0|_{g_{jk}=0}}{g_{jk}}. \qquad (7)$$

If g_{jk} is allowed to approach infinity, the last term vanishes, the first term on the right-hand side remains finite and nonzero, and the left-hand side is unaffected. Therefore,

$$\Delta_{Djk}^0 = \lim_{g_{jk}\to\infty}\left[\frac{\Delta_D^0}{g_{jk}}\right]. \qquad (8)$$

A similar analysis applies if row l and column m are deleted, so

$$\Delta_{Djk,lm}^0 = \lim_{g_{jk}\to\infty}\left[\frac{\Delta_{Dlm}^0}{g_{jk}}\right]. \qquad (9)$$

Thus

$$\frac{\Delta_{Djk,lm}^0}{\Delta_{Djk}^0} = \lim_{g_{jk}\to\infty}\left[\frac{\Delta_{Dlm}^0}{\Delta_D^0}\right] = R_{ml}^{jk} = \lim_{g_{jk}\to\infty} R_{ml}^0. \qquad (10)$$

It is possible to contrive circuits for which $\Delta_{Djk}^0 = 0$ and $a_{jk,lm} \neq 0$. In such cases, the relation $a_{jk,lm} = \lim_{g_{lm}\to\infty} R_{kj}^0/g_{jk}$ can be used to find $a_{jk,lm}$.

In general, calculation of R_{ml}^{jk} requires that the transresistance from port l to port m be calculated for a test circuit in which a voltage-controlled current-source (VCCS) of value $g_x V_k$ is placed at port j. The resulting expression is then evaluated in the limit $g_x \to \infty$. Fortunately, with many models, such a VCCS already exists in parallel with each transcapacitor, so the expression for R_{ml}^0 already includes the needed terms for g_{jk}, and no additional test circuit need be evaluated. For the case of a reciprocal capacitor C_{jj}, R_{ml}^{jj} is found by computing R_{ml}^0 for a circuit with port j shorted.

Note that the roles of index pairs lm and jk are completely symmetrical, so the coefficient of $C_{jk}C_{lm}$ can be computed in either of two ways:

$$a_{jk,lm} = R_{ml}^0 R_{jk}^{lm} = R_{ml}^{jk} R_{jk}^0. \qquad (11)$$

The second-order coefficient can be computed using

$$a_2 = \sum_{C_{D2}} C_{jk,lm} a_{jk,lm}. \qquad (12)$$

The third-order term is based on the set C_{D3} of all unique products of triplets $C_{jk}C_{lm}C_{pq}$ of nonzero capacitive elements in C_D such that $j \neq l$, $l \neq p$, and $p \neq j$, and $k \neq m$, $m \neq q$, and $q \neq k$. The restrictions eliminate triplets with two or more elements in the same row (a capacitor and a transcapacitor or two transcapacitors at the same port), and those with elements in the same column (a capacitor and a TCV or two TCV's at the same port). Each triplet has a corresponding coefficient, which is easily shown to be

$$a_{jk,lm,pq} = R_{kj}^0 R_{ml}^{jk} R_{qp}^{jk,lm} = a_{jk,lm} R_{qp}^{jk,lm} \qquad (13)$$

where $R_{qp}^{jk,lm} = \lim_{\substack{g_{jk}\to\infty \\ g_{lm}\to\infty}} [R_{qp}^0]$. The products of all the triplets in C_{D3} and their corresponding coefficients can be summed to form a_3.

This procedure can then be generalized and used to calculate denominator terms through a_{N_d}. The order of the denominator is usually less than N_d, since many of the higher order terms are zero.

Determination of the Numerator Polynomial

The numerator of the system function can be written as

$$\frac{\Delta_N}{\Delta_D^0} = \frac{\Delta_{io}}{\Delta_D^0} = h_0 + h_1 s + h_2 s^2 + \cdots h_N s^N \qquad (14)$$

where N is the order of the numerator.

The constant term h_0 is the dc limiting value of the transfer function

$$h_0 = \frac{\Delta_{io}^0}{\Delta_D^0} = H^0 = \lim_{s\to 0} H(s). \qquad (15)$$

The first-order coefficient h_1 is based on the set C_N of nonzero capacitive elements C_{jk}, $j \neq i$, and $k \neq o$. The restrictions on j and k eliminate any elements that form part of a reverse signal path with a transcapacitor at the input port or with a TCV at the output. Also excluded are capacitors in parallel with the input or output ports. Usually most of the elements of C_N are also elements of C_D. For each such element C_{jk}, the corresponding coefficient is

$$h_{jk} = \frac{\Delta_{io,jk}^0}{\Delta_D^0} = \frac{\Delta_{io,jk}^0}{\Delta_{Djk}^0}\frac{\Delta_{Djk}^0}{\Delta_D^0} = \left[\lim_{g_{jk}\to\infty}[H^0]\right]R_{kj}^0 = H^{jk} a_{jk}. \qquad (16)$$

For the case $j = k$, $g_{jk} = g_{jj} \to \infty$ implies that port j is to be shorted.

Now C_N may contain elements for which $a_{jk} = \Delta_{Djk}^0 = 0$. C_N may also contain elements not present in C_D, corresponding to elements of the form C_{ji} or C_{ok}, eliminated when the input or output was shorted to find the denominator. For these elements, which form parts of unilateral signal paths, (16) cannot be applied. In these cases, however, h_{jk} can be found another way.

TABLE II
SUMMARY OF TRANSFER FUNCTION COEFFICIENT CALCULATIONS FOR THE EXTENDED TIME-CONSTANT METHOD

Coefficient	Conditions
$a_{jk} = R^{0}_{kj}$	
$a_{jk,\,lm} = R^{lm}_{kj}\, a_{lm} = R^{jk}_{ml}\, a_{jk}$	$j \neq l,\ k \neq m$
$a_{jk,\,lm,\,pq} = R^{jk,lm}_{qp}\, a_{jk,\,lm}$	$j \neq l,\ j \neq q,\ l \neq q,\ k \neq m,\ k \neq p,\ m \neq p$
⋮	
$h_0 = H(s)\vert_{s \to 0} = H^0$	
$h_{jk} = H^{jk}\, a_{jk}$ $h_{jk,\,lm} = H^{jk,\,lm}\, a_{jk,\,lm}$ ⋮	$C_{jk} \in C_D$, and $a_{jk} \neq 0$ C_{jk} and $C_{lm} \in C_D$, and $a_{jk} \neq 0$
$h_{jk} = \lim_{g_{jk} \to \infty} H^0/g_{jk}$ $h_{jk,\,lm} = \lim_{g_{jk} \to \infty,\ g_{lm} \to \infty} H^0/(g_{jk} \cdot g_{lm})$ ⋮	C_{jk} not $\in C_D$, or $a_{jk} = 0$ C_{jk} or C_{lm} not $\in C_D$, or $a_{jk,lm} = 0$

By analogy to (9), $\Delta_{io,jk}$ can be written as

$$\Delta_{io,jk} = \lim_{g_{jk} \to \infty} \left[\frac{\Delta_{io}}{g_{jk}} \right]. \tag{17}$$

If $a_{jk} = \Delta^0_{Djk} / \Delta^0_D = 0$, from (17), $\Delta^0_D = \Delta^0_D|_{g_{jk}=0}$, which means that Δ^0_D does not depend on g_{jk}. Thus $\lim_{g_{jk} \to \infty} \Delta^0_D = \Delta^0_D$ is finite. This is equally true if C_{jk} was eliminated from set C_D. In either case,

$$h_{jk} = \frac{\Delta_{io,jk}}{\Delta^0_D} = \lim_{g_{jk} \to \infty} \left[\frac{\Delta_{io}}{\Delta^0_D g_{jk}} \right] = \lim_{g_{jk} \to \infty} \left[\frac{H^0}{g_{jk}} \right]. \tag{18}$$

Once all of the first-order coefficients have been found, h_1 can be found by summing the $h_{jk} C_{jk}$ products over the whole set:

$$h_1 = \sum_{C_N} h_{jk} C_{jk}. \tag{19}$$

The second-order coefficient is based on the set C_{N2} of all unique products of pairs $C_{jk} C_{lm}$ of nonzero capacitive elements in C_N such that $j \neq l$ and $k \neq m$. If $\Delta^0_{Djk,lm}$ is nonzero and both C_{jk} and C_{lm} are elements of C_D,

$$h_{jk,lm} = \lim_{\substack{g_{jk} \to \infty \\ g_{lm} \to \infty}} [H^0] R^{jk}_{ml} R^0_{kj} = H^{jk,lm} R^{jk}_{ml} R^0_{kj} = H^{jk,lm} a_{jk,lm}. \tag{20}$$

If C_{jk} or C_{lm} is not an element of C_D, or if $a_{jk,lm} = 0$, then

$$\lim_{\substack{g_{jk} \to \infty \\ g_{lm} \to \infty}} \Delta^0_D = \Delta^0_D$$

so that

$$h_{jk,lm} = \lim_{\substack{g_{jk} \to \infty \\ g_{lm} \to \infty}} \left[\frac{H^0}{g_{jk} g_{lm}} \right]. \tag{21}$$

Generalization to higher order coefficients is straightforward. The results of this section are summarized in Table II.

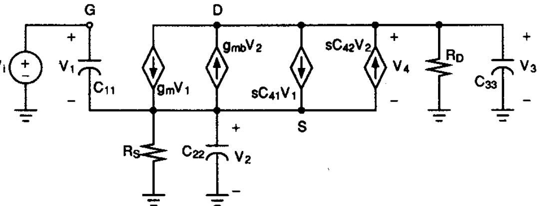

Fig. 1. Common-source amplifier equivalent circuit.

TABLE III
TRANSFER CALCULATION FOR FIG. 1

Coefficient	How Computed	Result, where $D = 1 + (g_m + g_{mb}) R_S$
a_{11}	R^0_{11}	R_S/D
a_{22}	R^0_{22}	R_S/D
a_{33}	R^0_{33}	R_D
a_{41}	R^0_{14}	R_S/D
a_{42}	R^0_{24}	R_S/D
$a_{11,\,22}$	$R^0_{22} R^{22}_{11}$	0
$a_{11,\,33}$	$R^0_{33} R^{33}_{11}$	$R_D R_S/D$
$a_{11,\,42}$	$a_{42} R^{42}_{11}$	0
$a_{22,\,33}$	$a_{22} R^{22}_{33}$	$R_S R_D/D$
$a_{22,\,41}$	$a_{41} R^{41}_{22}$	0
$a_{33,\,41}$	$a_{41} R^{41}_{33}$	$R_S R_D/D$
$a_{33,\,42}$	$a_{42} R^{42}_{33}$	$R_S R_D/D$
$a_{11,\,22,\,33}$	$a_{11,\,22} R^{11,\,22}_{33}$	0
$a_{11,\,33,\,42}$	$a_{11,\,42} R^{11,\,42}_{33}$	0
$a_{22,\,33,\,41}$	$a_{22,\,41} R^{22,\,41}_{33}$	0
h_0	H^0	$-g_m R_D/D$
h_{11}	$a_{11} H^{11}$	$+g_{mb} R_D R_S/D$
h_{22}	$a_{22} H^{22}$	$-g_m R_D R_S/D$
h_{41}	$a_{41} H^{41}$	$-g_m R_D R_S/D$
h_{42}	$a_{42} H^{42}$	0
$h_{11,\,22}$	$\lim_{g_{11} \to \infty,\ g_{22} \to \infty} H^0/(g_{11} \cdot g_{22})$	0
$h_{11,42}$		0
$h_{22,\,41}$	$\lim_{g_{22} \to \infty,\ g_{41} \to \infty} H^0/(g_{22} \cdot g_{41})$	0

III. EXAMPLE: COMMON-SOURCE AMPLIFIER

An example will help to clarify the extended time-constant technique. The circuit shown in Fig. 1 represents a common-source amplifier with source degeneration. The transistor model includes two transcapacitors, C_{41} and C_{42}, in parallel with g_m and g_{mb} VCCS's. The circuit with all capacitive elements removed is analyzed to find the voltage transfer function. The results are given in Table III.

All of the capacitive elements contribute to the first-order time constant. Coefficient $a_{41} = R^0_{14}$ is found by applying a current source I_4 at port 4 in the same direction as the C_{41} controlled source; R^0_{41} is found as V_2/I_4. a_{42} is found by a similar technique.

There are 10 unique combinations of the five capacitive elements taken in pairs; of these, pairs $C_{11}C_{41}$, $C_{22}C_{42}$, and $C_{41}C_{42}$ are excluded from the set C_{D2} since they each share common ports. The remaining second-order coefficients are easy to evaluate. $a_{11,22} = a_{11}R_{22}^{11}$ is obviously zero, since shorting port 1 also reduces the resistance at port 2 to zero. Coefficients $a_{22,41} = a_{41}R_{22}^{41}$ and $a_{11,42} = a_{42}R_{22}^{42}$ are also both zero, since $R_{22}^{41} = \lim_{g_m \to \infty} R_{22}^{0}$ and $R_{11}^{42} = \lim_{g_{mb} \to \infty} R_{11}^{0}$ are both zero. Similarly, R_{33}^{41} can be found as $\lim_{g_m \to \infty} R_{33}^{0}$ and R_{33}^{42} is $\lim_{g_{mb} \to \infty} R_{33}^{0}$, both of which are just R_D. Therefore,

$$\begin{aligned} a_{33,41} &= a_{41}R_{33}^{41} = a_{33,42} \\ &= a_{42}R_{33}^{42} = R_D R_S / [1 + (g_m + g_{mb})R_S]. \end{aligned} \tag{22}$$

Similar analysis shows that $a_{11,33}$ and $a_{22,33}$ give the same result.

Although there are 10 ways to form triplets of the capacitive elements, all but three are eliminated from set C_{D3} because of shared ports. The remaining coefficients all evaluate to zero.

The leading coefficient h_0 of the numerator polynomial is the dc value of the transfer function:

$$h_0 = H^0 = -g_m R_D / [1 + [g_m + g_{mb})R_S]. \tag{23}$$

The set C_N needed to find the first-order coefficient h_1 includes only the four elements C_{11}, C_{22}, C_{41}, and C_{42}; C_{33} is excluded since it is in parallel with the output port. Most of the terms needed to find h_1 can be calculated by inspection. For example, H^{42} is just $\lim_{g_{mb} \to \infty} H^0$ so that $h_{42} = 0$.

There are three pairs of elements in set C_{N2}: $C_{11}C_{22}$, $C_{11}C_{42}$, and $C_{22}C_{41}$. Since h_{42} is zero, $h_{11,42}$ is zero. In the other two cases, since the corresponding denominator coefficient is zero, (21) must be used. Since H^{41} is finite, $h_{22,41}$ can be found from

$$\begin{aligned} h_{22,41} &= \lim_{\substack{g_{22} \to \infty \\ g_{41} \to \infty}} \left[\frac{H^0}{g_{22}g_{41}} \right] = \lim_{R_S \to 0} \left[\frac{R_S H^{41}}{g_m} \right] \\ &= \lim_{R_S \to 0} \left[\frac{R_S R_D}{g_m} \right] = 0. \end{aligned} \tag{24}$$

The complete transfer function can be written as

$$\frac{V_o(s)}{V_i(s)} = -\frac{[g_m + (g_m C_{22} + g_m C_{41} - g_{mb} C_{11})R_S s] R_D}{A + Bs + Cs^2} \tag{25}$$

where

$$\begin{aligned} A &= 1 + (g_m + g_{mb})R_S \\ B &= R_S\{C_{11} + C_{22} + C_{41} + C_{42} + C_{33}[1 + (g_m + g_{mb})R_D]\} \\ C &= R_D R_S (C_{11}C_{33} + C_{22}C_{33} + C_{33}C_{41} + C_{33}C_{42}). \end{aligned}$$

IV. Conclusion

This work has demonstrated an extension of the time-constant method to allow for nonreciprocal capacitive elements. A large number of cross products often arise in the higher order terms of transfer functions. Adding transcapacitors to the transistor model complicates the expressions even more. For this reason, the most common use of the extended time-constant method is likely to be to provide an easy way to gain an intuitive feel for the effects of transcapacitive elements on circuit behavior.

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Some Schur-Stability Criteria for Uncertain Systems with Complex Coefficients

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***Abstract* —Let a discrete-time uncertain system be characterized by a polynomial $f(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n$, with complex coefficients $a_i = x_i + jy_i$ whose real and imaginary parts belong to some known intervals. The stability of such systems, in the face of coefficient variation, is under question. In this paper, three approaches are presented for robust stability analysis of these kinds of systems.**

I. Introduction

Let a set of polynomials of degree n having complex coefficients be presented by

$$f(z) = \sum_{i=0}^{n} a_i z^{n-i} \tag{1.1}$$

where in general, for $0 \leqslant i \leqslant n$, $a_i = x_i + jy_i$ $(x_0 + jy_0 \neq 0)$, and x_i and y_i are arbitrary but fixed in the real intervals:

$$x_i \in [\underline{x}_i, \bar{x}_i]; \qquad y_i \in [\underline{y}_i, \bar{y}_i]. \tag{1.2}$$

The problem of Schur robust stability of (1.1) when a_i's are real ($y_i = 0$, for all i, $0 \leqslant i \leqslant n$) has been tackled by several researchers. Hollot and Bartlett [1] reported that if real coefficients a_i's are constant for $i = 0, 1, \cdots, n/2\rfloor$ (where by $n/2\rfloor$ we mean the next lowest integer with respect to $n/2$), then the interval polynomial $f(z)$ is Schur if and only if all the corner polynomials corresponding to $a_i \in \{\underline{a}_i, \bar{a}_i\}$, $i = n/2\rfloor + 1, \cdots, n$ are Schur. This result was generalized in [2] for the case when all the real coefficients a_i are subject to change. There are counterexamples [1] that even the "weak" version of Kharitonov's theorem, which states that for continuous-time systems a neces-

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